De Bruijn Graph and Sequence

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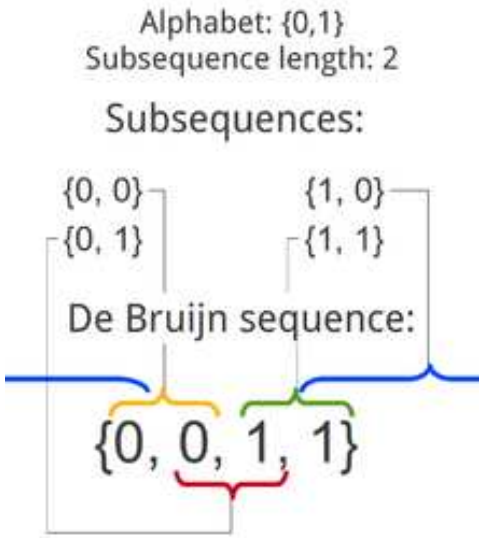
De Bruijn graph and sequence

named after the Dutch mathematician **Nicolaas Govert de Bruijn**.



The De Bruijn sequence $B(k, n)$ is a **cyclic sequence** of a given alphabet A with size k for which every possible subsequence of length n appears as a sequence of consecutive characters exactly once.

Constructing a De Bruijn Sequence

To construct a sequence $B(k, n)$ we compute an Eularian cycle of a $(n-1)$-dimensional De Bruijn graph over k symbols.

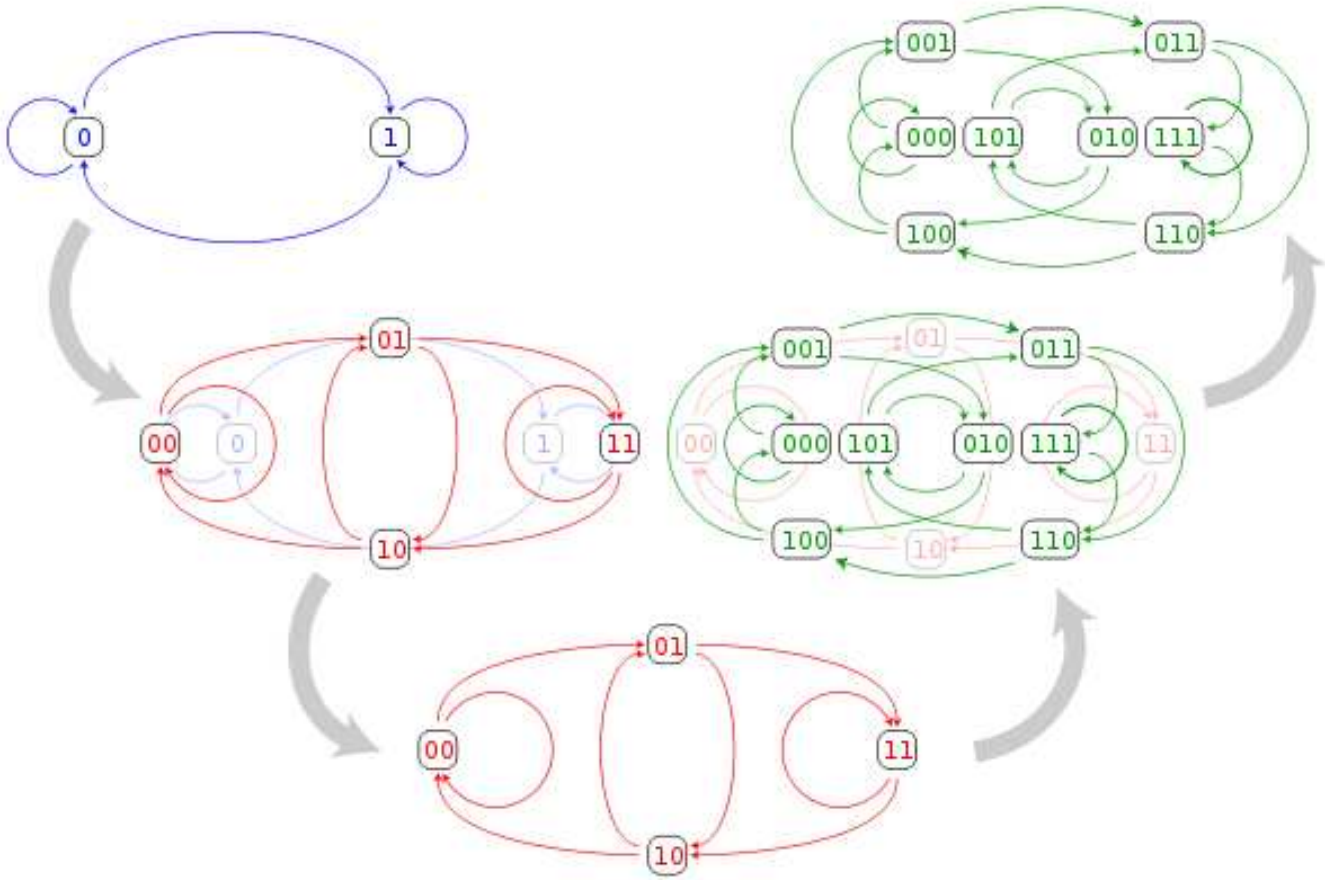
In graph theory, an n-dimensional **De Bruijn graph** of k symbols is a directed graph representing overlaps between sequences of symbols. It has k^n vertices, consisting of all possible length-n sequences of the given symbols.

A directed edge is drawn from vertex $v_1 = (s_i ... s_{i+n-1})$ to $v_2 = (s_j ... S_{j+n})$ if the symbol at s_{i+1} is identical to s_j, the symbol at s_{i+2} is identical to s_{j+1}, ... and the symbol at s_{i+n-1} is identical to s_{j-1+n}.

De Bruijn Graph Properties

- if $n = 1$ all the vertices are connected forming a total of k^2 edges
- each vertex has exactly k incoming and k outgoing edges
- each n-dimensional De Bruijn graph is the line digraph of the $(n-1)$-dimensional De Bruijn graph with the same set of symbols
- each De Bruijn graph is **Eulerian** (it has an Eularian circuit, visiting every **edge** exactly once)
- each De Bruijn graph is **Hamiltonian** (it has an Hamiltonian path, visiting every **vertex** exactly once)
- the Euler cycles and Hamiltonian cycles of these graphs are De Bruijn sequences

De Bruijn Graph Properties



Each vertex of the n-dimensional De Bruijn graph corresponds to an edge of the $(n-1)$-dimensional De Bruijn graph, and each edge in the n-dimensional De Bruijn graph corresponds to a two-edge path in the $(n-1)$-dimensional De Bruijn graph.

De Bruijn Graph for all possible nucleotide doublets