ADS: Algorithmen und Datenstrukturen 2

Teil VII: Suffix Trees

Steve Hoffmann

Bioinformatics Group, Dept. of Computer Science & Interdisciplinary Center for Bioinformatics, **University of Leipzig**

21. Mai 2010

suffix and prefix: intuition

What is a suffix?

- "*suffix?*" is a suffix of "*What is a suffix?*"
- "*a suffix?*" is a suffix of "*What is a suffix?*"
- "*What is a suffix?*" for sure is a suffix of "*What is a suffix?*"

What is a prefix?

- "*What*" is a prefix of "*What is a suffix?*"
- "*What is*" is a prefix of "*What is a suffix?*"
- "*What is a suffix?*" for sure is a prefix of "*What is a suffix?*"

formalities: alphabet and characters

Let $\mathcal{A}$ be a finite set, the alphabet.

- the elements of $\mathcal{A}$ are characters.
- ϵ denotes the empty string.
- strings are written by juxtaposition of characters:

The set $\mathcal{A}^*$ of strings over $\mathcal{A}$ is defined by

$$\mathcal{A}^* = \bigcup_{i \geq 0} \mathcal{A}_i \tag{1}$$

where $\mathcal{A}_0 = \{\epsilon\}$ and $\mathcal{A}_{i+1} = \{aw \mid a \in \mathcal{A}, w \in A^i\}$.

- $\mathcal{A}^+$ denotes a non-empty string

formalities: strings, suffixes and prefixes

Let s be a string over the alphabet $\mathcal{A}$

- let $|s|$ denote the length of s.

We assume a string of the form $s = uvw,\ u, v, w \in \mathcal{A}^*$:

- u is a prefix of s
- v is a substring of s
- w is a suffix of s

Note, that by using ϵ every string may be partioned to uvw!

Searching strings

Current genome projects and internet data bases accumulate massive amounts of sequences (texts). Searching such texts can be pretty cumbersome!

Considering a large sequence S over $\mathcal{A} = \{\mathtt{A}, \mathtt{C}, \mathtt{T}, \mathtt{G}\}$. We may ask:

- does ACTGCTTACGTACGGTA occur in S?
- how often does ACTGCTTACGTACGGTA occur in S?
- where does ACTGCTTACGTACGGTA occur in S?

For large sequences that are frequently queried we need more sophisticated strategies to answer these questions quickly.

Indexing

For a large string s that is frequently queried (ie. a genome) we could think about **indexing all substrings**. The following theorem shows why this is **not such a good idea**:

Theorem

A string s of length $|s| = n$ has at most $O(n^2)$ different substrings.

Proof.

Idea: for each $0 \leq i \leq n-1$,

$$\bigcup_{i \leq j \leq n-1} s[i..j] \tag{2}$$

is the set of substrings beginning at position i. Alphabet? $\square$

Hence, we need another way to represent this data.

towards suffix trees: implicit representation of substrings

Consider the string $s = abab$

string	a	b	a	b
position	0	1	2	3

we observe substrings:

a ab aba $abab$ b ba bab

- **substrings are prefixes of suffixes**: $abab$, bab, ab, b
- each substring is implicitly represented by at least one suffix
- **at most** $O(n^2)$ **substrings but only** On **suffixes!**

towards suffix trees: implicit representation of substrings

Consider the string $s = abab$

string	a	b	a	b
position	0	1	2	3

we observe substrings:

$a \quad ab \quad aba \quad abab \quad b \quad ba \quad bab$

- **substrings are prefixes of suffixes**: $abab$, bab, ab, b
- each substring is implicitly represented by at least one suffix
- **at most** $O(n^2)$ **substrings but only** On **suffixes!**

towards suffix trees: implicit representation of substrings

Consider the string $S = abab$

we observe substrings:

$a \quad ab \quad aba \quad abab \quad b \quad ba \quad bab$

substrings are prefixes of suffixes:

$abab, \ bab, \ ab, \ b$

- each substring is implicitly represented by at least one suffix
- **at most $O(n^2)$ substrings but only n suffixes!**

towards suffix trees: implicit representation of substrings

Consider the string $S = abab$

we observe substrings:

$a \quad ab \quad aba \quad abab \quad b \quad ba \quad bab$

substrings are prefixes of suffixes:

$abab,\ bab,\ ab,\ b$

- each substring is implicitly represented by at least one suffix
- **at most $O(n^2)$ substrings but only n suffixes!**

towards suffix trees: implicit representation of substrings

Consider the string $S = abab$

- we observe substrings a, ab, aba, $abab$, b, ba and bab
- **substrings are prefixes of suffixes**: $abab$, bab, ab, b
- each substring is implicitly represented by at least one suffix
- **at most** $O(n^2)$ **substrings but only** n **suffixes!**

We append a unique character (sentinel) \$ to the end of s:

$$S\$ = abab\$$$

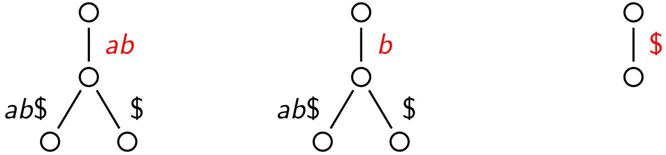
Reason?

towards suffix trees: common prefixes

Given all suffixes *abab*\$, *bab*\$, *ab*\$, *b*\$, \$ we observe

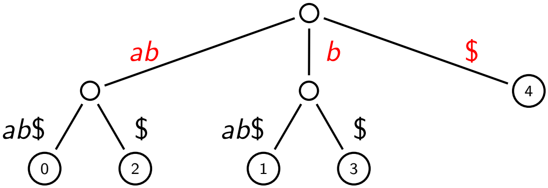
- *ab* is longest common prefix for *abab*\$ and *ab*\$
- *b* is longest common prefix for *bab*\$ and *b*\$
- \$ is longest prefix of \$

This gives us three partial trees:



a suffix tree

Combining the three partial trees leads to a complete suffix tree:



string	a	b	a	b	$
position	0	1	2	3	4

More on suffix tree construction later ...

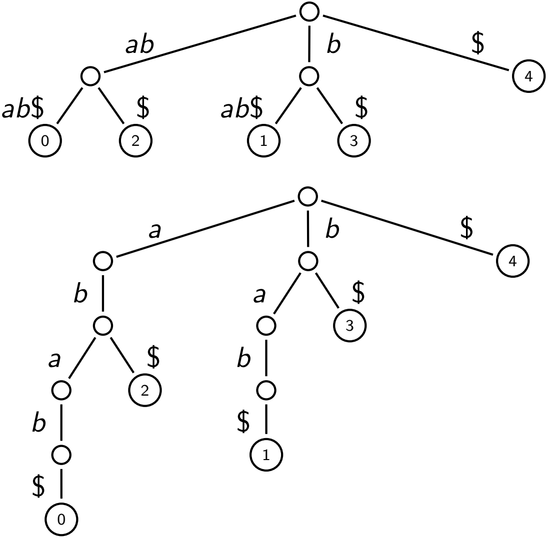
formalities: suffix tree

Definition (suffix tree)

A suffix tree $ST(S\$)$ for a sequence $S \in \mathcal{A}^+$

1. edges labeled by non-empty strings over $\mathcal{A}$
2. for every node only one edge begins with same $a \in \mathcal{A}$
3. compact, ie. has no internal nodes with less than two successors (in contrast to suffix trie)
4. represents all substrings of S

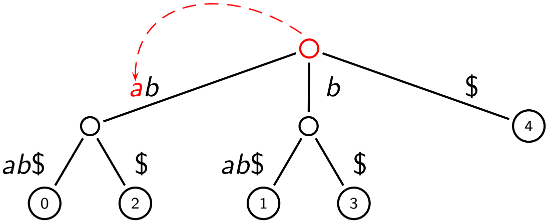
a suffix tree is compact, a suffix trie is not



searching in a suffix tree

Searching a query sequence q in a suffix tree is easy. At internal nodes we look for a branch that starts with next character in q, on edges we simply continue to match the labels.
Lets search the query $q = ab$.

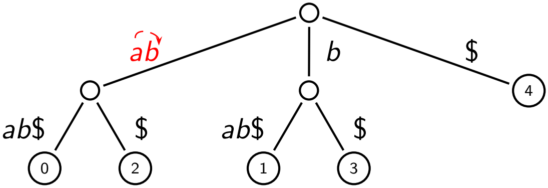
- the search starts with the first character of q, $q[0] = a$



searching in a suffix tree

Searching a query sequence q in a suffix tree is easy. At internal nodes we look for a branch that starts with next character in q, on edges we simply continue to match the labels.
Lets search the query $q = ab$.

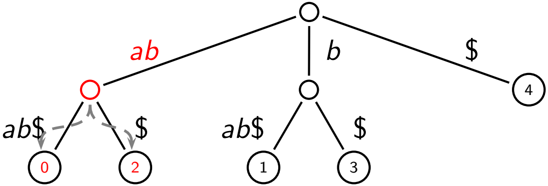
- the search starts with the first character of q, $q[0] = a$
- and continues with the next character $q[1] = b$



searching in a suffix tree

Searching a query sequence q in a suffix tree is easy. At internal nodes we look for a branch that starts with next character in q, on edges we simply continue to match the labels.
Lets search the query $q = ab$.

- the search starts with the first character of q, $q[0] = a$
- and continues with the next character $q[1] = b$
- we immediately see: q occurs twice in $S\$$ at positions 0 and 2.



complexity considerations

The suffix tree $ST(S)$ for the sequence $S\$$ with $|S| = n$

- hast at most $n - 1$ internal nodes and n leaves $\Rightarrow O_{space}(n)$

Due to their compactness suffix trees require much less space in practice compared to suffix tries.

- for a query of length m, searches in the suffix tree take at most m character comparisons $\Rightarrow O_{search}(m)$

naive construction: top-down

In order to devise an algorithm to construct suffix trees, we will make use of the following observations:

- a node α represents all suffixes of $S\$$ that start with the same prefix u
- only one outgoing edge from α begins with same $a \in \mathcal{A}$
- we can restrict our evaluation to at most $|\mathcal{A}|$ subsets of suffixes with uav, where $a \in \mathcal{A}$ and $v \in \mathcal{A}^*$

naive construction: simplifications

We will now devise the algorithm **topdown**. For reasons of simplicity we assume

- To construct the suffix tree we call **topdown** $(U, 0)$, where U is a set of pairs that initially(!) holds all suffixes and their positions in S\$, ie.
  $U = \{(s, i) \mid S\$[i..n-1] = s, 0 \leq i \leq n-1\}$
- let **getlcp(U)** be a function evaluates the longest common prefix for all strings s in $(s, i) \in U$
- let **insertEdge(**α**,** u**,** β**)** be a function that inserts an edge with label u between the nodes α and β.
- let **insertLeaf(**α**,** u**,** i**)** be a function that attaches a leaf i with edge label u to α.

naive construction: pseudo-code

algorithm: topdown

Require: set of pairs U, length of lcp ℓ

    $\forall a \in \mathcal{A} : P_a = \emptyset$, node α
    for all $(s, i) \in U$ **do**
        $a := s[\ell]$
        $P_a := P_a \cup (s[\ell + 1..n - 1], i)$
    end for
    for all $a \in \mathcal{A}$ **do**
        if $|P_a| > 1$ **then**
            $v :=$ getlcp(P_a)
            $\beta :=$ topdown$(P_a, |v|)$
            insertEdge(α, av, β)
        else if $|P_a| = 1$ **then**
            $(s, i) := P_a$
            attachLeaf(α, as, i)
        end if
    end for
    return α

naive construction: example

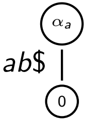
For $s = abab\$$: $U = \{(abab\$, 0), (bab\$, 1), (ab\$, 2), (b\$, 3), (\$, 4)\}$

- $P_a = \{(bab\$, 0), (b\$, 2)\}$, $\text{getlcp}(P_a) = 1$

naive construction: example

For $s = abab\$$: $U = \{(abab\$, 0), (bab\$, 1), (ab\$, 2), (b\$, 3), (\$, 4)\}$

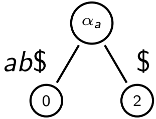
- $P_a = \{(bab\$, 0), (b\$, 2)\}$, $\text{getlcp}(P_a) = 1$
  - $P_a = \{(b\$, 0)\} \longrightarrow \text{addLeaf}(\alpha_a,\ ab\$,\ 0)$



naive construction: example

For $s = abab\$$: $U = \{(abab\$, 0), (bab\$, 1), (ab\$, 2), (b\$, 3), (\$, 4)\}$

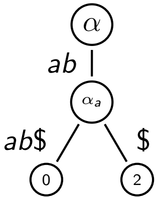
- $P_a = \{(bab\$, 0), (b\$, 2)\}$, $\text{getlcp}(P_a) = 1$
  - $P_a = \{(b\$, 0)\} \longrightarrow \text{addLeaf}(\alpha_a,\ ab\$,\ 0)$
  - $P_\$ = \{(\epsilon, 2)\} \longrightarrow \text{addLeaf}(\alpha_a,\ \$,\ 2)$



naive construction: example

For $s = abab\$$: $U = \{(abab\$, 0), (bab\$, 1), (ab\$, 2), (b\$, 3), (\$, 4)\}$

- $P_a = \{(bab\$, 0), (b\$, 2)\}$, $\text{getlcp}(P_a) = 1$
  - $P_a = \{(b\$, 0)\} \longrightarrow \text{addLeaf}(\alpha_a,\ ab\$,\ 0)$
  - $P_\$ = \{(\epsilon, 2)\} \longrightarrow \text{addLeaf}(\alpha_a,\ \$,\ 2)$
- $\longrightarrow \text{insertEdge}(\alpha,\ ab,\ \alpha_a)$



Can you continue this example?

naive construction: complexity

The naive construction does not perform very well

- complexity of naive suffix tree construction is $O(n^2)$. Proof?

We can do better:

- Ukkonen has devised an online construction algorithm that runs in O(n).
- McCreight's O(n)-algorithm also runs slightly faster in practice.

McCreight's algorithm

- The idea: successively merge $ST(S\$[i..n])$ with $ST(S\$[i+1..n])$, $i \in [0, n]$
- exploits the properties of longest common prefixes

Definition (head)

The *head(i)* of some suffix $S\$[i..n]$, $0 \le i \le n-1$ is the longest common prefix of with $S\$[j..n]$, where $j < i$.

Definition (tail)

The *tail(i)* is simply the rest: $S\$[i..n] = head(i)tail(i)$

heads up: McCreight's trick

To appreciate the trick we need to understand the following observation:

1. Once the suffix tree for $S\$[0..n]..S\$[i..n]$ is build, we need to **update only those parts of ST that are “affected“** by introducing the suffix $S\$[i+1..n]$
2. Only parts beyond the head, ie. within the tail, may be “affected“.

heads up: McCreight's trick

Beweis.

Let ST(S) be as suffix tree of a string S. Assume the string av has a prefix implicitly represented in $ST(S)$. Hence, there is a search-path from the root to some edge or node (leaf?)
$\Rightarrow$ the search-path is of length $max_{j \leq |S|} lcp(av, S[j..n]) = \ell$
$\Rightarrow$ $lcp(v, S[j..n]) = k \geq \ell - 1$
$\Rightarrow$ substring $v[0...\ell-1]$ is also implicitly represented by ST(S)
$\Rightarrow$ only suffix tree modifications below internal nodes or edge labels representing $v[\ell-1..n]$ are necessary □

Hence, finding an inexpensive way to locate the head yields an inexpensive algorithm. Let's pretend we've found it:

magic(head(i)) delivers $p \in ST$ with p representing the substring $S\$[i+1..i+\ell-1]$.

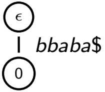
linear construction: algorithm M

```
Require: inital suffix tree for S$[0..n-1]
  for i in 0 to n − 2 do
    if head(i) = ϵ then
      head(i+1) := scan(ϵ, S$[i+1..n])
      if head(i+1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if p ∈ E then
        head(i+1) = p
      else if p ∈ V then
        head(i+1) = scan(p, tail(i))
      end if
      if head(i + 1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for
```

linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, S\$$[i{+}1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

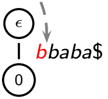
i	0
S[i+1..n]	baba$
head(i)	ϵ
tail(i)	bbaba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i{+}1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

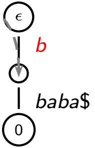
i	0
S[i+1..n]	baba$
head(i)	ϵ
tail(i)	bbaba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i{+}1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

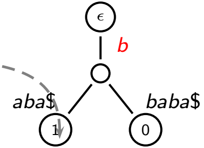
i	0
S[i+1..n]	baba$
head(i)	ϵ
tail(i)	bbaba$



linear construction: algorithm M

```
Require: inital suffix tree for S$[0..n-1]
  for i in 0 to n − 2 do
    if head(i) = ε then
      head(i+1) := scan(ε, S$[i+1..n])
      if head(i+1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if p ∈ E then
        head(i+1) = p
      else if p ∈ V then
        head(i+1) = scan(p, tail(i))
      end if
      if head(i + 1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for
```

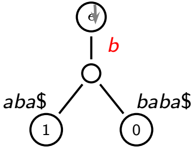
i	0
S[i+1..n]	baba$
head(i)	ϵ
tail(i)	bbaba$



linear construction: algorithm M

```
Require: inital suffix tree for S$[0..n-1]
  for i in 0 to n − 2 do
    if head(i) = ϵ then
      head(i+1) := scan(ϵ, S$[i+1..n])
      if head(i+1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if p ∈ E then
        head(i+1) = p
      else if p ∈ V then
        head(i+1) = scan(p, tail(i))
      end if
      if head(i + 1) ∈ E then
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for
```

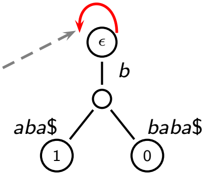
i	1
S[i+1..n]	aba$
head(i)	b
tail(i)	aba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i+1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

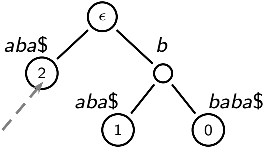
i	1
S[i+1..n]	aba$
head(i)	b
tail(i)	aba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i+1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

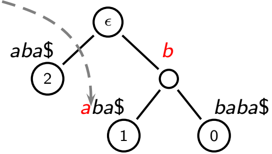
i	1
S[i+1..n]	aba$
head(i)	b
tail(i)	aba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
for i in 0 to $n-2$ **do**
  if head(i) $= \epsilon$ **then**
    head(i+1) := scan(ϵ, $S\$[i+1..n]$)
    if head(i+1) $\in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  else
    p := magic(head(i))
    if $p \in E$ **then**
      head(i+1) $= p$
    else if $p \in V$ **then**
      head(i+1) = scan(p, tail(i))
    end if
    if $head(i+1) \in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  end if
end for

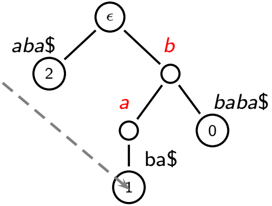
i	2
S[i+1..n]	ba$
head(i)	ϵ
tail(i)	aba$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i+1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

i	2
S[i+1..n]	ba$
head(i)	ϵ
tail(i)	aba$

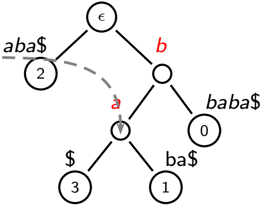


linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]

for i in 0 to $n-2$ **do**
  if head(i) $= \epsilon$ **then**
    head(i+1) := scan(ϵ, $S\$[i+1..n]$)
    if head(i+1) $\in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  else
    p := magic(head(i))
    if $p \in E$ **then**
      head(i+1) $= p$
    else if $p \in V$ **then**
      head(i+1) $=$ scan(p, tail(i))
    end if
    if $head(i+1) \in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  end if
end for

i	2
S[i+1..n]	ba$
head(i)	ϵ
tail(i)	aba$

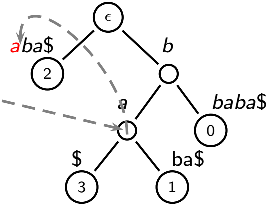


linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]

for i in 0 to $n-2$ **do**
  if head(i) $= \epsilon$ **then**
    head(i+1) := scan(ϵ, $S\$[i+1..n]$)
    if head(i+1) $\in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  else
    p := magic(head(i))
    if $p \in E$ **then**
      head(i+1) $= p$
    else if $p \in V$ **then**
      head(i+1) = scan(p, tail(i))
    end if
    if $head(i+1) \in E$ **then**
      addNode(head(i+1))
    end if
    addLeaf(head(i+1), tail(i+1), i+1)
  end if
end for

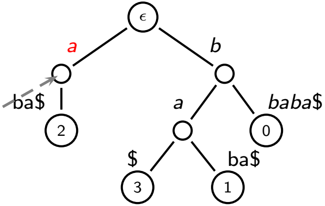
i	3
S[i+1..n]	a$
head(i)	ba
tail(i)	$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i+1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

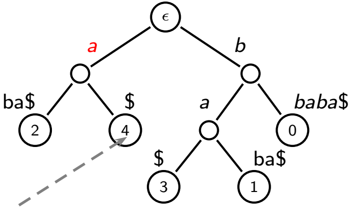
i	3
S[i+1..n]	a$
head(i)	ba
tail(i)	$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
  for i in 0 to $n-2$ **do**
    if head(i) $= \epsilon$ **then**
      head(i+1) := scan(ϵ, $S\$[i+1..n]$)
      if head(i+1) $\in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    else
      p := magic(head(i))
      if $p \in E$ **then**
        head(i+1) $= p$
      else if $p \in V$ **then**
        head(i+1) = scan(p, tail(i))
      end if
      if $head(i+1) \in E$ **then**
        addNode(head(i+1))
      end if
      addLeaf(head(i+1), tail(i+1), i+1)
    end if
  end for

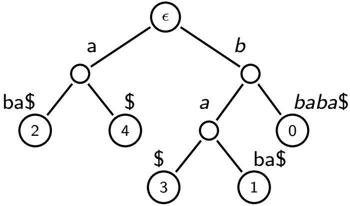
i	3
S[i+1..n]	a$
head(i)	ba
tail(i)	$



linear construction: algorithm M

Require: inital suffix tree for S$[0..n-1]
 for i in 0 to $n-2$ **do**
  if head(i) $= \epsilon$ **then**
   head(i+1) := scan(ϵ, $S\$[i+1..n]$)
   if head(i+1) $\in E$ **then**
    addNode(head(i+1))
   end if
   addLeaf(head(i+1), tail(i+1), i+1)
  else
   p := magic(head(i))
   if $p \in E$ **then**
    head(i+1) $= p$
   else if $p \in V$ **then**
    head(i+1) = scan(p, tail(i))
   end if
   if $head(i+1) \in E$ **then**
    addNode(head(i+1))
   end if
   addLeaf(head(i+1), tail(i+1), i+1)
  end if
 end for

i	4
S[i+1..n]	$
head(i)	a
tail(i)	$



What will happen now?

magic

For some head(i)=$S\$[i..i+\ell]$, the function **magic(head(i))** delivers a position $p \in ST(S)$ with p representing the substring $S\$[i+1..i+\ell-1]$. It is realized using **suffix links**:

- suffix links are only defined on nodes
- to use suffix link, go back to parent node
- while going back remember all edge labels
- jump to another node via suffix link
- rematch the edge labels
- done. we reached the location for $S\$[i+1..i+\ell-1]$.

Suffix links can be updated during the construction of the suffix tree with algorithm M.

Suffix trees: a stringology toolbox

1. string search
2. longest common substrings (two strings)
3. longest repeated substrings (one string)
4. for each suffix of a pattern, get length of the longest match
5. shortest unique substrings
6. ...